AIAA JOURNAL
Vol. 32, No. 10, October 1994

New Approach for Simple Prediction of Impact Force History on Composite Laminates

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A new method for simple prediction of the impact force history on composite laminates subjected to low-velocity impact is proposed. First, the impact force history is calculated from the impact response analysis using the finite element method incorporated with the modified Hertzian contact law. Frequency characteristics of the numerical impact force history are investigated from modal analysis and compared with the natural frequencies of the system in which the mass of an impactor is lumped with the plate. The present method can be efficiently applied to isotropic or orthotropic plates with unknown contact laws as well as composite laminates with no restraint on the material properties and the geometrical shape. Predicted impact force histories using the present method are compared with the numerical ones from the impact response analysis using the contact law as well as the experimental result using a drop weight type impact test system.

Nomenclature

c_l	= coefficient of sine function in linear approximate impact force history
c_{n1}, c_{n2}	= coefficients of sine function in geometrical nonlinear approximate impact force history
$(F_l)_{\max}$, $(F_n)_{\max}$	= maximum force in linear and geometrical nonlinear spring-mass model, respectively
$F_l(t)$, $F_n(t)$	= linear and nonlinear impact force history in approximate method, respectively
f_1, f_2	= first and second natural frequency of lumped mass system, respectively
k_l, k_n	= linear and nonlinear stiffness in spring-mass model, respectively
m	= mass in spring-mass model
m_P, m_I	= mass of plate and impactor, respectively
T_l, T_n	= first period of linear and nonlinear impact force history, respectively
v	= impact velocity of impactor
$(x_l)_{\max}$, $(x_n)_{\max}$	= maximum displacement in linear and nonlinear spring-mass model, respectively
ξ	= mass ratio between impactor and plate, m_I/m_P

Introduction

LOW-VELOCITY impact problems in composite structures have been a hot issue for the last two decades because damage due to the low-velocity impact might be left undetected and become potentially dangerous. Numerous analytical methods have been developed for solving this impact problem in composite laminates.[1,2]

Recently, to determine the contact force from the impact response analysis an experimental indentation law or the modified Hertzian contact law proposed by Yang and Sun[3] has been used by many researchers.[3–8] The dynamic response analyses using this contact (or indentation) law have provided many numerical results on the impact response. However, these analyses using the finite element method on the impact response of the laminate have been known to require long computation time even with modern computers. Furthermore, when the geometrical nonlinear analysis considering the large deflection effects of the laminate is performed, computation time becomes much longer, and the numerical scheme to trace the solution is very complex.[6,7]

Shivakumar et al.[9] did not use the finite element method to analyze the impact response of the laminate but used the spring-mass model to efficiently predict the impact force history. In their study the contact energy due to local indentation as well as the transverse shear energy and the bending energy of the plate are considered, and they reported that the contact energy can be neglected in the impact by relatively low velocity foreign object on a flexible or a thin plate. However, their study was restricted to only circular laminates with transversely isotropic material properties. Also, the governing equation is derived in the form of two coupled nonlinear differential equations which have to be solved using a particular numerical scheme.

In the present study, we propose a new method for simple prediction of the impact force history on composite laminates subjected to low-velocity impact. Since we consider the impact problem on thin plates, the contact energy is neglected in this study as it has been in other researches.[4–8] The impact duration is computed from the eigenvalue analysis of the lumped mass system in which the mass of an impactor is lumped with the plate, and the impulse-momentum conservation law is used with the concept of the spring-mass model.

Impact Force History Using Indentation Law

Finite Element Analysis and Numerical Results

First, the impact force history is calculated from the impact response analysis using the finite element method. This analytical

Table 1 Material properties of graphite/epoxy lamina and impactor

Lamina		
Stiffness	$E_1 = 135.4$ GPa, $G_{12} = G_{13} = 4.8$ GPa,	$E_2 = E_3 = 9.6$ GPa $G_{23} = 3.2$ GPa
Poisson's ratio	$\nu_{12} = \nu_{13} = 0.31$,	$\nu_{23} = 0.52$
Thickness	$h = 0.1125$ mm	
Density	$\rho = 1580.0$ kg/m^3	
Impactor		
Stiffness	$E = 207.0$ GPa	
Poisson's ratio	$\nu = 0.3$	
Local radius in contact	$r = 0.6$ cm	

Received Oct. 19, 1993; revision received April 4, 1994; accepted for publication April 8, 1994.

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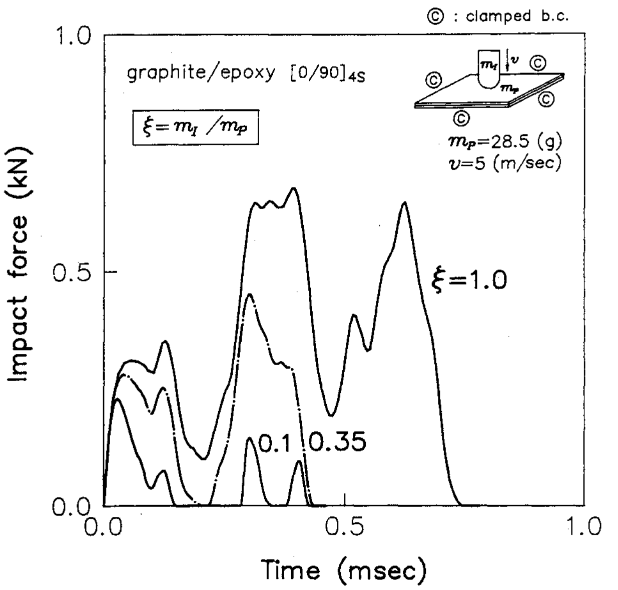

Fig. 1 Impact force histories from the finite element method using the contact law when $\xi \leq 1.0$ and $v = 5$ m/s.

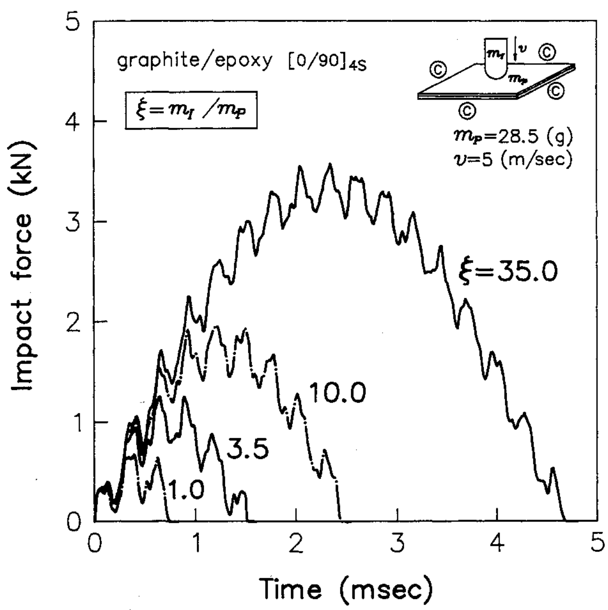

Fig. 2 Impact force histories from the finite element method using the contact law when $\xi \geq 1.0$ and $v = 5$ m/s.

method is the same as in Refs. 6 and 7 except that the modified Hertzian contact law is used in this analysis instead of the experimental indentation law. This analysis may be summarized as follows. The first-order shear deformation theory and von Kármán's large deflection theory are used to describe the dynamic response of the plate. The modified Hertzian contact law proposed by Yang and Sun,[3] which consists of three relations according to the loading, unloading, and reloading paths, is used to determine the contact force. The Newmark's constant acceleration method is used to solve the time-dependent equation and two successive iterations (one related with the geometrical nonlinear problem and the other to determine the contact force) are performed and continued until the desired accuracy is obtained in each time step. The response of the impactor is assumed to be rigid body motion.

In this study, a laminated plate made with Han Kuk Carbon Co., Ltd. CU-125 graphite/epoxy prepreg is considered. The impact test size of the laminate is $10 \times 10 \times 0.18$ cm, hence, it is a thin plate with the width-to-thickness ratio of about 56. The stacking sequence is $[0/90]_{4S}$ and the material properties of a lamina and the impactor are shown in Table 1. The boundary condition of the plate has four edges clamped. Since the plate is a cross-ply laminate only a quarter plate is analyzed. For finite element modeling using the nine-node isoparametric quadrilateral plate element, 8×8 rectangular mesh is used, which is estimated by the authors to be converged enough. As the time step in dynamic analysis, 5 ~ 15 μs is used according to the impact condition including the mass and the velocity of the impactor.

In Figs. 1 and 2, numerical impact force histories from the impact response analysis, without considering the geometrical nonlinear effects, are shown for a constant impact velocity $v = 5$ m/s, and for various mass ratios of the impactor and the plate, ξ.

Figure 1 are force histories when $\xi \leq 1.0$ and Fig. 2 when $\xi \geq 1.0$. In Fig. 1, when ξ is 0.1, the contact between impactor and the laminate occurs three times; when ξ is 0.35, it occurs two times,

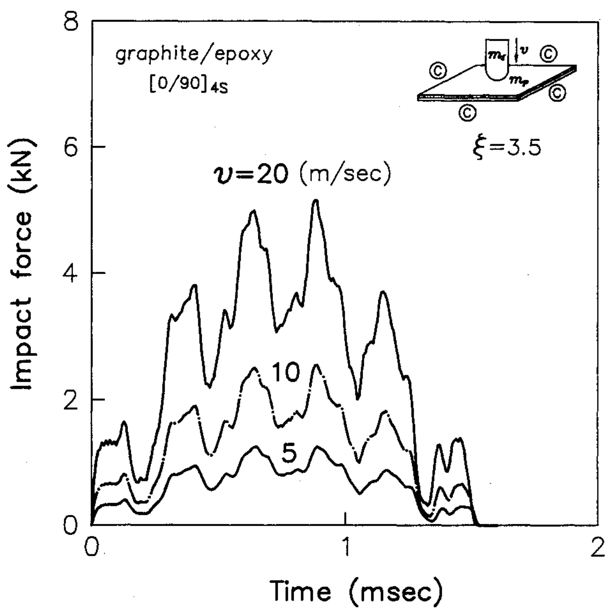

Fig. 3 Impact force histories from the finite element method using the contact law when $\xi = 3.5$ and impact velocity is varied.

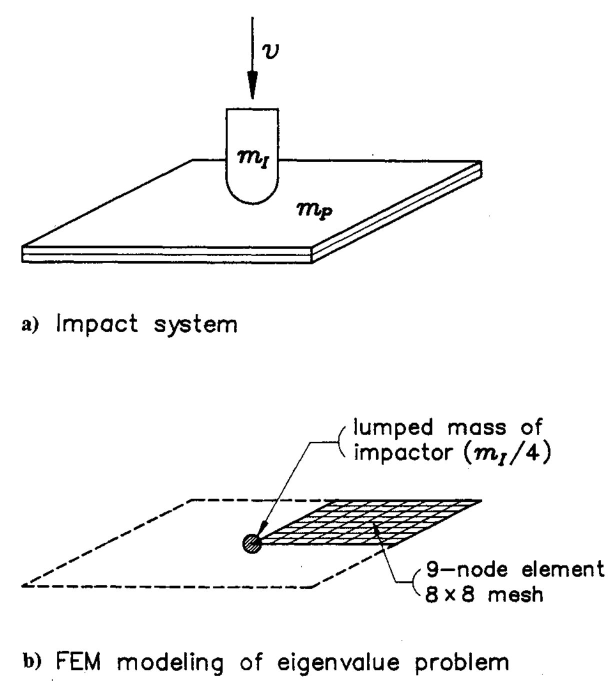

Fig. 4 Finite element modeling for eigenvalue analysis of the lumped mass system.

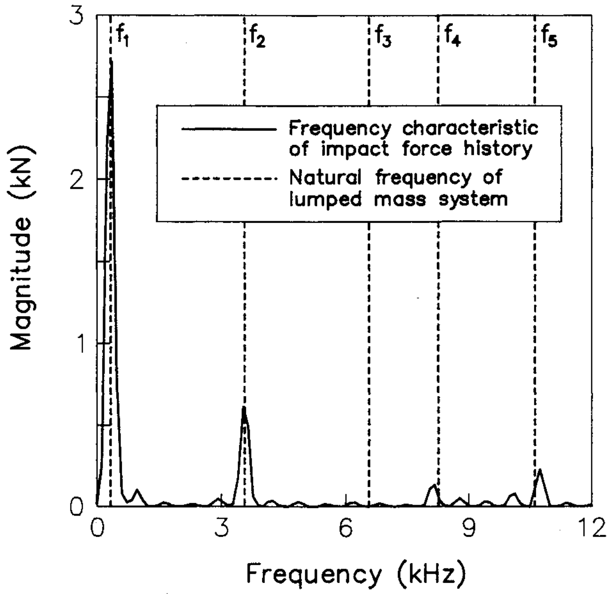

Fig. 5 Frequency characteristics of numerical impact force history and natural frequencies of the lumped mass system.

and when ξ is 1.0, it occurs only once. In Fig. 2, when ξ is larger than 1.0 the contact occurs only once. In summary, when ξ is larger than 1.0, contact occurs only once. In Fig. 2, as ξ becomes still larger, the contact duration also becomes larger. Also, the whole shape of the contact force history becomes increasingly similar to a sine wave, and the secondary fluctuations of the force history curve become smaller in comparison to the full shape of the curve.

In Fig. 3, the impact force histories are shown when the mass ratio is constant, i.e., $\xi = 3.5$ and the impact velocity is varied. As the velocity becomes larger, the force history becomes larger in proportion to the magnitude of the velocity, but the whole shape of the contact force history remains unchanged. Accordingly, it can be understood that the duration and the shape of the contact force history are not dependent on the velocity but on the mass ratio.

Frequency Characteristics

In Figs. 1 and 2, when the mass ratio is larger than 1.0 (i.e., when the mass of the impactor is larger than that of the laminate), it is shown that the contact force history continues during the impact duration with only one contact. This means that the impactor keeps on contacting the laminate during the impact duration. Consequently, it is obvious that the characteristics of the dynamic response including the contact force history of the impact system agree with those of the system when the impactor and the laminate becomes a single body.

To investigate the frequency characteristics of the numerical contact force history, MSA MODAL software, a program for modal analysis, was used. Also, to investigate the vibration characteristics of the impact system, the eigenvalue problem of the lumped mass system in which the mass of the impactor is lumped with the laminate was solved using the finite element method as shown in Fig. 4.

In Fig. 5, the frequency characteristics of the numerical contact force history are compared with the natural frequencies of the lumped mass system in a typical case where the mass ratio is 3.5 and the impact velocity is 10 m/s. The frequency characteristics agree well with the natural frequencies of the lumped mass system, and there are especially very good agreements in the first and the second frequency.

Simple Prediction on Impact Force History

Linear Analysis and Results

In present study, to predict the impact force history, the frequency characteristics examined in the last section are applied. We let the first period of the contact force history T_l (two times the impact duration) be the first natural period of the lumped mass system, i.e., $1/f_1$. This impact duration determined from the eigenvalue analysis may be more accurate than that from a direct analysis of the spring-mass model, as shown in Ref. 9 where the effects of the mass of the plate were simplified with a concept of the effective plate mass in the spring-mass model analysis.

In the linear analysis, the load-deflection curve is linear as shown in Fig. 6b. Applying the concept of the spring-mass model in Fig. 6a, the contact force history may be assumed to be a sine function as in Eq. (1).

$$F_l(t) = c_l \sin\frac{2\pi t}{T_l} \qquad \left(0 \le t \le \frac{T_l}{2}\right) \tag{1}$$

Neglecting thermal and acoustic dissipation as well as energy loss due to impact damage and residual vibration of the laminate after the impact, the mechanical energy of the impactor is conservative. Therefore, we may let the rebound velocity of the impactor be equal to the impact velocity. Applying the impulse-momentum conservation law to the impact force history of Eq. (1),

$$2m_l v = \int_0^{T_l/2} F_l(t)\, \mathrm{d}t \tag{2}$$

the approximate impact force history can be written as

$$F_l(t) = \frac{2\pi m_l v}{T_l} \sin\frac{2\pi t}{T_l} \qquad \left(0 \le t \le \frac{T_l}{2}\right) \tag{3}$$

In Fig. 7, the approximate impact force histories from Eq. (3) are shown. The contact durations agree very well with those from the impact response analysis using the contact law. Also, the approximate force history agrees well with the average value of the secondary fluctuation of the numerical impact force history curve. This approximate method provides more accurate results as the mass ratio becomes larger, because the secondary fluctuation of the numerical impact force history curve becomes smaller in comparison to the sine-wave form of variation of the full curve.

Figure 8 shows the relation between the relative magnitude of the secondary fluctuation in comparison to the full amplitude of

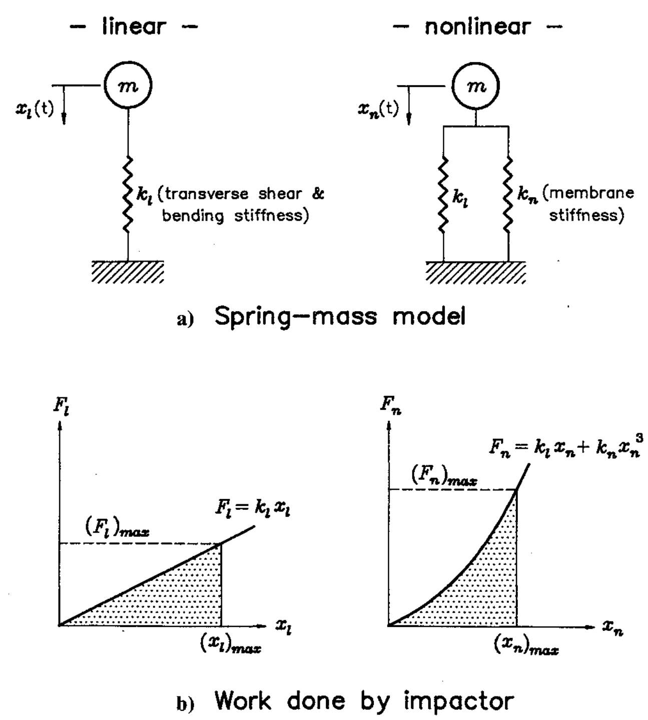

Fig. 6 Spring-mass model and the work done by impactor.

the numerical impact force history curve and the mass ratio. The relative magnitude of the secondary fluctuation becomes smaller as the mass ratio becomes larger. Figure 9 shows the relation between the natural frequency ratio of the lumped mass system (i.e., f_2/f_1) and the mass ratio. The natural frequency ratio becomes larger as the mass ratio becomes larger. From these two results, it can be understood that the error from the secondary fluctuation becomes smaller as the mass ratio becomes larger.

Figure 10 shows the approximate impact force histories when the mass ratio is large, i.e., $\xi = 35$. The present prediction provides good agreement with the numerical results with variation of the impact velocity.

From the results shown in Figs. 7–10, a simple summary can be written as follows. To predict the contact force history of the low-velocity impact problem using the present simple method, the natural frequency ratio between the second frequency and the first frequency of the lumped mass system in which the mass of the impactor is lumped with the laminate should be computed. If this frequency ratio is large enough, the impact force history using the present method will provide a good approximate result (if the frequency ratio is larger than 20, i.e., the mass ratio larger than 15, then the maximum error due to secondary fluctuation is less than 10%, as shown in Figs. 8 and 9).

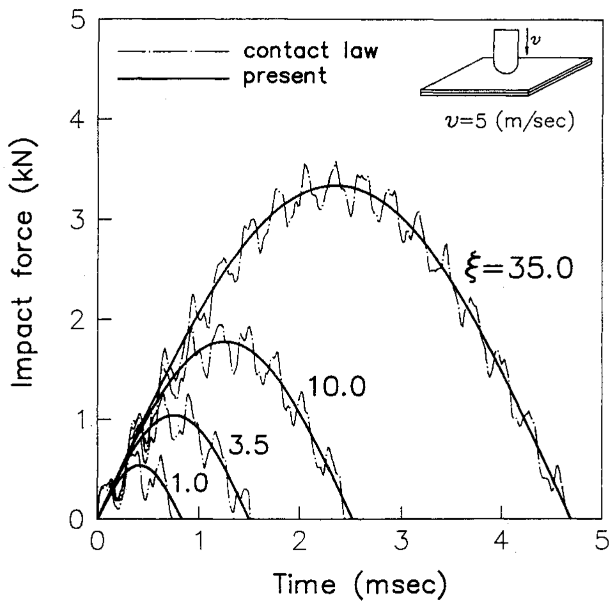

Fig. 7 Impact force histories using the contact law and present approximate results when ξ is varied and $v = 5$ m/s.

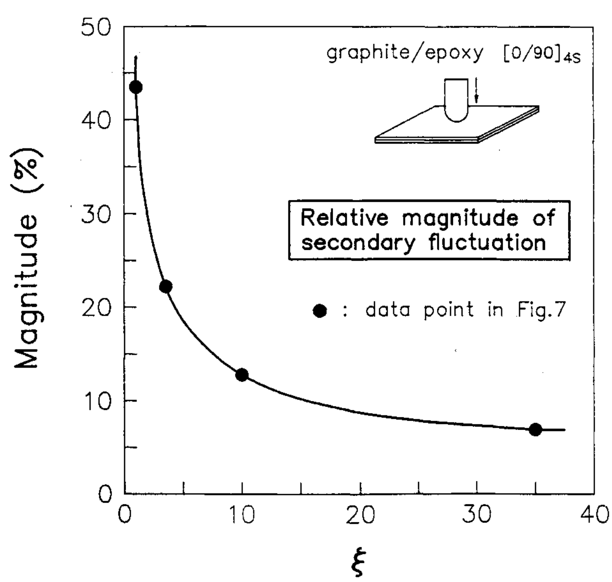

Fig. 8 Relationship between relative magnitude of secondary fluctuation of numerical impact force history curve and the mass ratio.

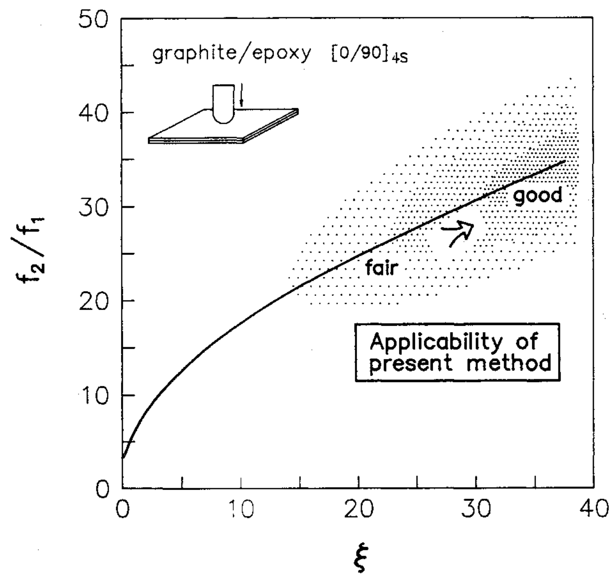

Fig. 9 Relationship between natural frequency ratio of lumped mass system and the mass ratio.

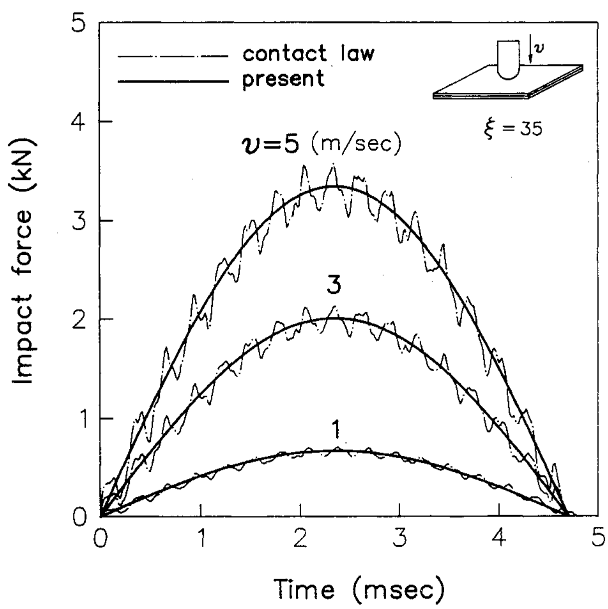

Fig. 10 Impact force histories using the contact law and present approximate results when $\xi = 35.0$ and impact velocity is varied.

Nonlinear Analysis and Results

To predict the impact force history by a geometrical nonlinear analysis to include the membrane effects due to the large deflection of the plate, the geometrical nonlinear eigenvalue problem should be solved by applying the concept of the lumped mass system as used in last section. Since the geometrical nonlinear eigenvalue problem of the laminate is a main subject itself, we did not try to directly solve this nonlinear problem. Instead we propose a new and simple method to obtain the nonlinear approximate impact force history as described in the following.

To consider the large deflection effects of the plate, the nonlinear stiffness k_n should be used as shown in Fig. 6a. In the nonlinear load-deflection relation, it is known that the load is proportional to the third order of the deflection.[9] Accordingly, the governing equation of the nonlinear spring-mass model can be written as

$$m\ddot{x}_n + k_l x_n + k_n x_n^3 = 0 \tag{4}$$

Equation (4) can be solved analytically using a series expansion of sine functions, so that the impact force history can be written in a series expansion as well. In this study, only the first two terms are considered, thus the approximate nonlinear impact force history can be assumed as shown in Eq. (5)

$$F_n(t) = c_{n1} \sin \frac{2\pi t}{T_n} + c_{n2} \sin \frac{6\pi t}{T_n} \qquad \left(0 \le t \le \frac{T_n}{2}\right) \tag{5}$$

where three unknowns, i.e., c_{n1}, c_{n2}, and T_n, should be determined, and, therefore, three equations are needed to determine the three variables.

The first equation is determined from the impulse-momentum law as shown in Eq. (6).

$$2m_I v = \int_0^{T_n/2} F_n(t)\, dt \tag{6}$$

The second equation, shown in Eq. (7), is given from the condition that the work done by the impactor is the same from both the linear and the nonlinear analyses.

$$\int_0^{(x_l)_{\max}} k_l x \, dx = \int_0^{(x_n)_{\max}} k_l x + k_n x^3 \, dx \tag{7}$$

When the deflection is small the large deflection effects are negligible, and so the initial slope of the impact force history in the nonlinear analysis is the same as that in the linear analysis. Therefore, the third equation is determined as shown in Eq. (8).

$$\left.\frac{dF_l(t)}{dt}\right|_{t=0} = \left.\frac{dF_n(t)}{dt}\right|_{t=0} \tag{8}$$

In the present study, the linear and the nonlinear stiffnesses of the laminate k_l and k_n are estimated by using the least square method from the numerical load-deflection curve which is obtained from the static analysis using the finite element method.

Solving these three equations by some elementary calculations, the nonlinear approximate impact force history is obtained as

$$F_n(t) = \frac{3\pi m_I v}{2T_n} \sin \frac{2\pi t}{T_n} + \frac{1}{4}(F_n)_{\max} \sin \frac{2\pi t}{T_n}$$

$$+ \frac{3\pi m_I v}{2T_n} \sin \frac{6\pi t}{T_n} - \frac{3}{4}(F_n)_{\max} \sin \frac{6\pi t}{T_n} \qquad \left(0 \le t \le \frac{T_n}{2}\right) \tag{9}$$

where T_n and $(F_n)_{\max}$ are given in Eqs. (10) and (11), and $(x_n)_{\max}$ and $(F_l)_{\max}$ are given in Eqs. (12) and (13).

$$(x_n)_{\max} = \left\{\sqrt{3\left(\frac{(F_l)_{\max}}{(F_n)_{\max}}\right)^2 + 1} - 1\right\} \tag{10}$$

$$(F_n)_{\max} = k_l (x_n)_{\max} + k_n (x_n)^3_{\max} \tag{11}$$

$$T_n = T_l \frac{(F_n)_{\max}}{(F_l)_{\max}} \left\{\sqrt{\left(\frac{k_l}{k_n}\right)^2 + \frac{2(F_l)^2_{\max}}{k_n k_l}} - \frac{k_l}{k_n}\right\}^{1/2} \tag{12}$$

$$(F_l)_{\max} = \frac{2\pi m_I v}{T_l} \tag{13}$$

In Fig. 11, the nonlinear approximation of the impact force history given by Eq. (9) and the numerical one from the nonlinear impact response analysis using the contact law as well as the linear analysis results are shown. There are good agreements between the approximate results and the numerical results except for a small difference due to the secondary fluctuation of the numerical curves. Also, from this result it can be understood that the linear analysis underestimates the impact force history and overestimates the contact duration.

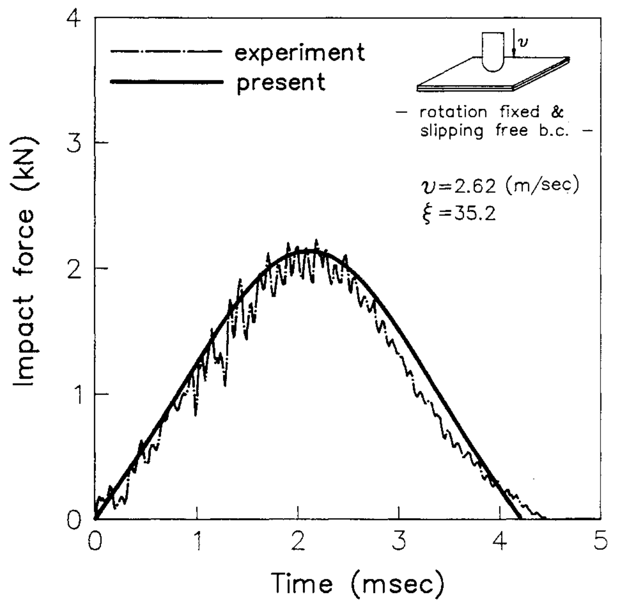

Fig. 12 Comparison of approximate impact force history with the experimental result when ξ = 35.2 and v = 2.62 m/s.

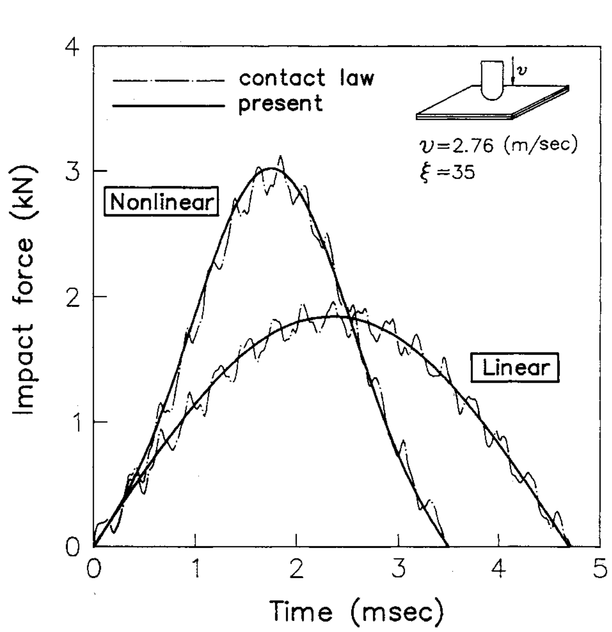

Fig. 11 Impact force histories using the contact law and present approximate results when ξ = 35.0 and v = 2.76 m/s.

Comparison with Experimental Result

To compare with the experimental result, a drop weight type impact test system set up by the authors is used. The mass of the impactor assembly weighs 1.00 kg, and the geometrical size and the material properties of the specimen and the impactor are the same as that described in the impact response analysis using the finite element method, as shown in Table 1. The four edges of the impact test specimen are clamped by bolts, then it is fixed to rotate at the edges. Some slipping at the edges may occur since it is impossible to restrain completely the in-plane displacements by only frictional forces from the clamped bolts. The maximum slipping displacement at the edge is estimated to be about 0.1 mm from numerical analysis under static loading of 2 kN, which is similar to the maximum impact force under the present impact test condition. Thus, we carefully conclude that even though it may be impossible to restrain perfectly this small displacement with only clamped bolts, it is believed that it may be possible by perfectly bonding the specimen to the specimen fixture, which would be the subject of another research.

In Fig. 12, the nonlinear approximate impact force history at the boundary condition which is rotation fixed and slipping free at all edges, agrees well with the experimental result. However, a small difference appears between the two results since the energy loss due to thermal and acoustic dissipation as well as impact damage is included in the experimental response as well as the rebound velocity (which is assumed to be equal to the impact velocity in the present approximate analysis). But from the viewpoint of simple prediction of the impact force history, the present method can be applied efficiently and accurately.

Conclusion

In this paper, a new method for simple prediction of the impact force history on composite laminates subjected to low-velocity impact is proposed. This method can be applied when the contact between the foreign object and the plate continues during the impact duration (which is in most cases when the mass of the object is larger than that of the plate).

The approximate impact force histories using the present method agree well with the numerical ones from the geometrical nonlinear impact response analysis as well as the linear analysis using the modified Hertzian contact law. This approximate result becomes more accurate as the mass ratio becomes larger, because the secondary fluctuations of the numerical impact force history curve become smaller in comparison with the whole variation of that curve. Also, a typical approximate result from the present nonlinear analysis shows a good agreement with the experimental result using a drop weight type impact test system.

In the present method, the impact duration is computed from the eigenvalue analysis of the lumped mass system, and the impulse-momentum conservation law is used with the concept of the spring-mass model to predict the impact force history. Also, because the contact energy due to local indentation in thin plates can be negligible, this method need not introduce the contact law. Consequently, it can be efficiently applied for a simple prediction of the low-velocity impact force history with no restraint of the material properties and the geometrical shape as well as the contact characteristics between the impactor and the plate.

Acknowledgment

The authors wish to thank the Korea Agency for Defence Development for the support of this study (Project ADD-90-5-002).

References

[1]Abrate, S., "Impact on Laminated Composite Materials," *Applied Mechanics Review*, Vol. 44, No. 4, 1991, pp. 155–190.

[2]Cantwell, W. J., and Morton, J., "The Impact Resistance of Composite Materials—A Review," *Composites*, Vol. 22, No. 5, 1991, pp. 347–362.

[3]Yang, S. H., and Sun, C. T., "Indentation Law for Composite Laminates," *Composite Materials: Testing and Design*, 6th Conference, American Society for Testing and Materials, ASTM STP 787, Philadelphia, PA, 1982, pp. 425–449.

[4]Tan, T. M., and Sun, C. T., "Use of Statistical Indentation Laws in the Impact Analysis of Laminated Composite Plates," *Journal of Applied Mechanics*, Vol. 52, March 1985, pp. 6–12.

[5]Sun, C. T., and Chen, J. K., "On the Impact of Initially Stressed Composite Laminates," *Journal of Composite Materials*, Vol. 19, Nov. 1985, pp. 490–504.

[6]Chen, J. K., and Sun, C. T., "Dynamic Large Deflection Response of Composite Laminates Subjected to Impact," *Composite Structures*, Vol. 4, No. 1, 1985, pp. 59–73.

[7]Lee, I., Choi, I. H., and Hong, C. S., "Low-Velocity Impact Response of Composite Laminates Using Large Deflection Theory," *Proceedings of the 7th International Conference on Composite Materials* (Guangzhou, China), Vol. 2, Nov. 1989, pp. 241–246.

[8]Wu, H. Y., and Springer, G. S., "Impact Induced Stresses, Strains, and Delamination in Composite Plates," *Journal of Composite Materials*, Vol. 22, June 1988, pp. 533–560.

[9]Shivakumar, K. N., Elber, W., and Illg, W., "Prediction of Impact Force and Duration Due to Low-Velocity Impact on Circular Composite Laminates," *Journal of Applied Mechanics*, Vol. 52, Sept. 1985, pp. 674–680.